Exercise 40

Find the derivative. Simplify where possible.

$$y = \sinh^{-1}(\tan x)$$

Solution

Take the derivative using the chain rule.

$$\begin{aligned} y' &= \frac{d}{dx}[\sinh^{-1}(\tan x)] \\ &= \frac{1}{\sqrt{1+(\tan x)^2}} \cdot \frac{d}{dx}(\tan x) \\ &= \frac{1}{\sqrt{1+\tan^2 x}} \cdot (\sec^2 x) \\ &= \frac{1}{\sqrt{\sec^2 x}} \cdot (\sec^2 x) \\ &= \frac{1}{|\sec x|} \cdot (|\sec x||\sec x|) \\ &= |\sec x| \end{aligned}$$